MATH-955 General Relativity and Cosmology

Credit Hours: 3-0

Prerequisite: None

Course Objectives: General Relativity (GR) is a physical theory of gravitation invented by Albert Einstein in the early twentieth century. The theory has strong mathematical setup, has immense predictive power, and has successfully qualified several experimental/observational experiments of astrophysics and cosmology. Black holes and relativistic cosmology are two main applications of GR. It is intended that GR and its major applications and achievements be discussed in the manner they deserve.

Core Contents: Special relativity revisited, Electromagnetism, The gravitational field equations, The Schwarzschild geometry, Schwarzschild black holes, Kerr metric, Further spherically symmetric geometries.

Detailed Course Contents: Special relativity revisited: Minkowskispacetime in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four- vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle, Four- momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowskispacetime in arbitrary coordinates.

Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge, Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.

The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of theEinstein equations, The cosmological- constant term.

The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff’s theorem, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles, Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits, Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.

Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation, Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of blackhole evaporation.

Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution, Radial photon trajectories in RN geometry, radial massive particle trajectories.

Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts). The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3- space, Friedmann- Robertson-Walker metric, geometrical properties of FRW metric, The cosmological redshift, The Hubble and deceleration parameters, Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmologicalmodels.

Learning Outcomes: Students will understand of the theory and predictions of Einstein's general relativity. Students will be capable to read research papers and initiate research in general relativity. Students will be able to understand the dynamical evolution of the universeby studying cosmology.

Text Book: M.P. Hobson, G.P. Efstathiou, A.N. Lasenby, General Relativity, Cambridge University Press (2007).

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	5.1-5.7	Special relativity revisited: Minkowski space time in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four- vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle.
2	5.8-5.14	Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowski space time in arbitrary coordinates.
3	6.1-6.4	Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge.
4	6.5-6.7	Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
5	8.1-8.7	The gravitational field equations: The energy–momentum tensor, The energy– momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of the Einstein equations, The cosmological-constant term.
6	9.1-9.7	The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff's theorem, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles.

7	9.8-9.13	Circular motion of massive particles, stability of massive particle orbits, trajectoriesof photons, circular motion of photons, stability of photon orbits.
8	10.1, 10.2, 10.4	Experimental tests of general relativity: Precession of planetary orbits, The bendingof light, Accretion discs around compact objects.
9	**Mid Semester Exam**	
10	11.1 - 11.6	Schwarzschild black holes: singularities in Schwarzschild metric, radial photonworldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation.
11	11.7 - 11.11	Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of blackhole evaporation.
12	12.1-12.6	Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution
13	12.7-12.8 13.5, 13.6	Radial photon trajectories in RN geometry, radial massive particle trajectories, Kerrmetric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts).
14	14.1-14.7	The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3-space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric.
15	14.9, 14.10	The cosmological redshift, The Hubble and deceleration parameters.
16	15.1-15.6	Components of the cosmological fluid, Cosmological parameters, The cosmologicalfield equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmological models.
17	-	Review
18	**End Semester Exam**	